The shear loading initially causes matrix damage, followed by fiber failures in the ply experiencing compression in the fiber direction. The orthotropy in material properties decreases in this ply relative to the other ply, creating shear-extension coupling. This results in further fiber damage in the ply and an eventual decrease in the final failure load. The results presented in Tables 1–3 corroborate this physical reasoning. The fiber damage produced by the damage-induced anisotropy is a small fraction of the fiber damage produced by the applied tension/compression loading for cases with high ratios of tension/compression to shear loading. For ratios greater than about 0.5, the difference between S_{xy} and S'_{xy} is relatively small, as one would expect.

The effects of loading rate on laminate behavior are shown in Figs. 1 and 2. Predicted stress–strain curves of $[\pm 30]_s$ laminate under shear for four loading rates are shown in Fig. 1a (solid, dashed, and dot–dashed lines correspond to ε_{xx}, ε_{yy}, and γ_{xy}, respectively). Variation of relative shear-extension coupling coefficients is shown in Fig. 1b (solid lines, A_{16}; dashed lines, A_{26}). The increase in loading rate substantially increases the strength of the laminate (see discussion in Ref. 1), but the values of A_{16}/A_{66} and A_{26}/A_{66} ratios at failure are independent of the loading rate. Comparisons with stress–strain diagrams predicted without accounting for damage-induced anisotropy (Fig. 2a) show that the effect of the damage-induced coupling on the shear strength is nearly independent of stress rate. However, variation of coupling coefficients under complex in-plane loading $\sigma_{xx} = \sigma_{yy} = \tau_{xy} = \sigma$ (Fig. 2b) depends on the stress rate. The coupling is lower at high loading rates.

The effect of stress deviation on damage-induced anisotropy in laminate under shear (Table 4) shows that maximal coupling at failure starts to be reduced at deviations higher than 10 MPa. The damage-induced coupling significantly decreases shear strength at low deviations. For example, neglecting damage-induced anisotropy almost doubles the estimate of shear strength at $\sigma_q = 0.1$ MPa.

Conclusions

The shear-extension coupling due to damage in an initially balanced $[\pm 30]_s$ laminate was studied using a damage evolution model developed by the authors. The effects of complex loading application and loading rate and deviation were investigated. It was shown that the combination of shear loading with uniaxial or biaxial tension and compression reduces shear-extension coupling when the longitudinal-to-shear-loading ratio is sufficiently high. Damage-induced shear-extension coupling is more pronounced at low loading deviations.

The analysis shows that neglecting damage-induced coupling, as most existing models do, may lead to substantial overestimation of the laminate strength. The overestimation by not taking into account the damage-induced anisotropy is severe for loads with a low deviation and predominantly shear loads. The cases studied in this Note clearly bring out the shortcoming of models that do not take into account the damage-induced anisotropy, especially when damage is slowly progressing. The effect of loading rate on angle-ply laminate was compared with previous experimental observations. Experiments studying the effect of all of the parameters discussed in this Note are not available in the open literature. The authors plan to carry out such experiments in the future to verify the predictive capabilities of the analytical model.

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Simplified Method for Predicting Onset of Open-Mode Free Edge Delamination

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Introduction

INTERLAMINAR stresses in composite laminates are caused by the mismatch in material properties of the constituent laminas and the stacking sequence.[1] Several methods are available for analyzing interlaminar stresses at free edges. These include the finite element method,[2] complex stress potential method,[3] and variational approach.[4] These solution procedures are all quite involved.

The interlaminar normal stress is caused by the mismatch in Poisson's ratios of the constituent laminas. The problem of predicting interlaminar normal stress and the influence of stacking sequence on laminate strength was discussed by Pagano and Pipes.[5,6] An approximate distribution of interlaminar normal stress at the free edge of a laminate in uniform axial strain was assumed based on the moment equilibrium of the upper sublaminate. Herakovich[1] discussed the relationship between engineering properties and delamination of composite materials. Interface moment was related to the resistance of the laminate to open-mode delamination. However, no equation describing this relation has been devised.

In this study, an attempt was made to relate average interlaminar normal stress to interface moment at the free edge for several families of laminates. By establishing a relation between these two variables, the average interlaminar normal stress was obtained by the classical laminated plate theory (CLPT). The relation was then used to predict the onset of open-mode free edge delamination for three laminates. The method was verified by comparing the predictions with existing experimental data.

Preliminary Consideration

Consider a balanced and symmetric laminate subjected to uniform axial extension along the x direction (Fig. 1). For a long laminate, the state of stress can be regarded as independent of the longitudinal direction, thus reducing the problem to a pseudo-three-dimensional problem.

The stress state in a sublaminate above the kth interface is illustrated in Fig. 2. Because of the mismatch in Poisson's ratios of the individual layers, transverse stresses σ_y are induced in these layers. The magnitudes and signs of these stresses depend only on the mismatch of the Poisson's ratios and can be determined by CLPT. They are independent of the stacking sequence. The equilibrium of this sublaminate in the y direction requires that $\tau_{yz}^k(y)$ be developed at the interface to balance the transverse stress. Moment equilibrium about the x axis requires that $\sigma_z^k(y)$ be developed at the interface; $\sigma_z^k(y)$ must be self-equilibrating, and its distribution must be equivalent to a couple of on-zero moments when equilibrium in the z direction is considered. Thus, interlaminar normal stress $\sigma_z^k(y)$ can be related to the in-plane transverse stresses $\sigma_y^i (i = 1, 2, \ldots, k)$ through

$$\int_0^b \sigma_z^k(y)\, y \,\mathrm{d}y = \sum^{k} \sigma_y^i h_i \left(\frac{z_i + z_{i-1}}{2} - z_k \right) = m^{(k)} \qquad (1)$$

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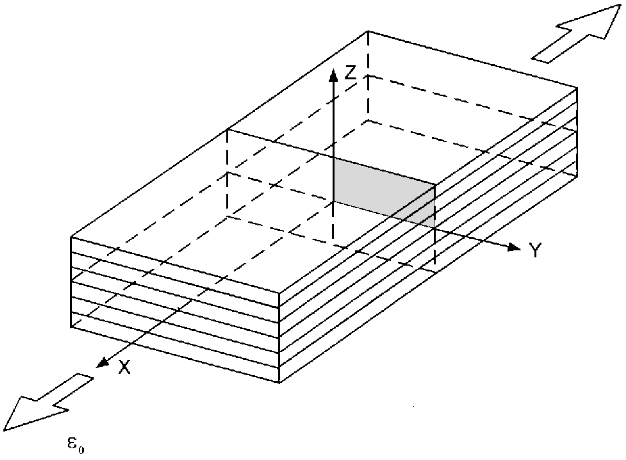

Fig. 1 Laminate subjected to uniaxial loading.

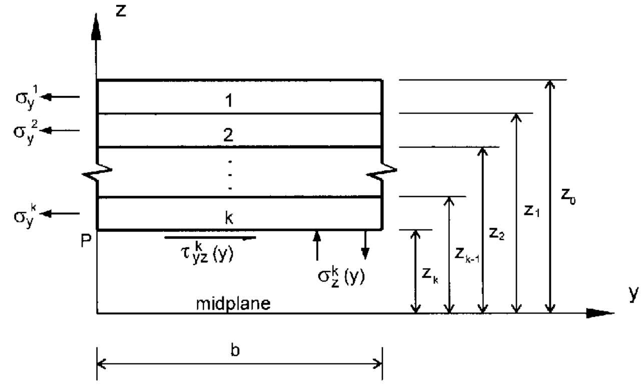

Fig. 2 Stress state in a sublaminate above the *k*th interface.

where $m^{(k)}$ is called the interface moment or unbalanced moment at the kth interface, z_i is the coordinate of the ith interface, h_i is the thickness of the ith ply, and b is the half-width of the laminate. The actual distribution of $\sigma_z^k(y)$ can only be determined by very involved analytical methods or finite element analysis.

Because interlaminar stress gradients in the vicinity of the free edge are very large (singular stresses exist at the edge), stress concentration factors are not adequate to characterize the stress levels that are responsible for interlaminar failure. It is customary to define an average stress by averaging the interlaminar stress component over a fixed distance (critical length) d_0 from the free edge.[7] For instance, the average interlaminar normal stress at the kth interface is defined by

$$\bar{\sigma}_z^{(k)} = \frac{1}{d_0}\int_{b-d_0}^{b} \sigma_z^k(y)\,\mathrm{d}y \tag{2}$$

In this study, the critical distance d_0 was chosen to be two times the ply thickness.[7] It is the objective of this study to relate this average stress to the interface moment $m^{(k)}$. Once this relationship is established, it is then possible to use CLPT to find $\bar{\sigma}_z^{(k)}$, which is subsequently used to predicate the onset of free edge delamination.

Relationship Between Average Interlaminar Normal Stress and Interface Moment

For a symmetric and balanced laminate under uniform axial strain ε_0, the pseudo-three-dimensional displacement field can be written in the form[2]

$$u = \varepsilon_0 x + U(y, z), \qquad v = V(y, z), \qquad w = W(y, z) \tag{3}$$

where u, v, and w are displacements in the x, y, and z directions, respectively; $U(y, z)$, $V(y, z)$, and $W(y, z)$ are functions of y and z only.

A finite element program[7] developed based on the preceding displacement field was used to calculate interlaminar stresses. The finite element model uses nine-node isoparametric elements to model a quadrant of the cross section. Very fine meshes were used near the edge. The material properties of AS4/3501-6 graphite/epoxy system in Ref. 7 were used in the computation.

The laminates considered are all eight-layer laminates including $[\pm\theta/0/90]_S$ families, $[\pm\theta/90_2]_S$ families, and the $[\pm 30/\pm 60]_S$ family, where $\theta = 30$, 45, and 60 deg. For each family, all possible stacking sequences were considered. For example, for the $[\pm 45/0/90]_S$ family, $[45/-45/0/90]_S$, $[45/0/90/-45]_S$, $[0/45/90/-45]_S$, etc., were analyzed individually. There are 24 possible stacking sequences for this family. However, only 12 of them are distinct because of the interchangeability of the +45- and −45-deg layers.

For each laminate, the interface moment at each interface was calculated by using CLPT and Eq. (1); average interlaminar normal stress at each interface was calculated by the finite element model and Eq. (2). Generally, in a laminate family, $[\pm 45/0/90]_S$, for example, a larger average interlaminar normal stress corresponds to a larger interface moment. When the average interlaminar normal stresses were plotted against the corresponding interface moments for each family, a very good linear relationship was observed. Figure 3 shows the result for the $[\pm 45/0/90]_S$ family. Each point represents the average interlaminar stress-interface moment relation at an interface. The solid line is the least square fit of the data points. The correlation coefficient is $R^2 = 0.987$.

Note that for $[\pm\theta/0/90]_S$ and $[\pm\theta/90_2]_S$ families with θ fixed, the linear relations are almost the same. This phenomenon suggests that, for a $[\pm\theta/0/90]_S$ or $[\pm\theta/90_2]_S$ family, the linearity is primarily determined by the $+\theta$- and $-\theta$-deg layers. For the $[\pm 30/\pm 60]_S$ family, the +30- and −30-deg layers determine the linearity of the relationship between average interlaminar normal stress and interface moment. This leads to the data points of the $[\pm 30/\pm 60]_S$ family being approximately coincident with those of the $[\pm 30/0/90]_S$ family and the $[\pm 30/90_2]_S$ family.

It is noted that for various θ, the slopes of the linear relations are slightly different. To establish a unified relation between the average interlaminar normal stress and the interface moment for all of the laminate families considered, an adjustment is needed. It is found that the slope of linear relation for each family is proportional to $[\sqrt[6]{(E_x/E_0)}]$, where E_x is the apparent Young's modulus of the $+\theta$- or $-\theta$-deg layer, and E_0 is the longitudinal Young's modulus of the unidirectional laminate. Thus, taking this value as an adjusting factor K defined by

$$K = (E_x/E_0)^{\frac{1}{6}} \tag{4}$$

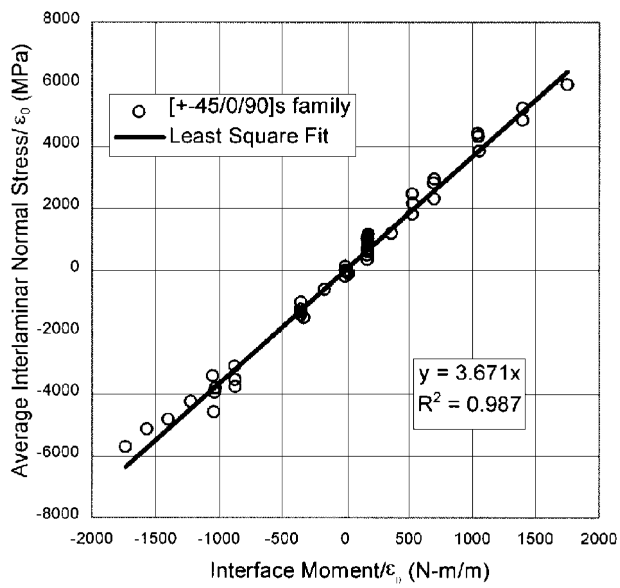

Fig. 3 Average interlaminar normal stress vs interface moment in $[\pm 45/0/90]_S$ family.

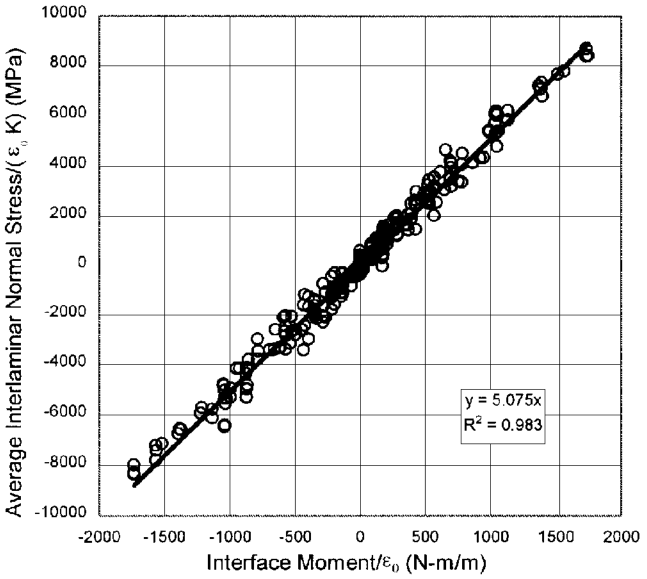

Fig. 4 Adjusted average interlaminar normal stress vs interface moment in $[\pm\theta/0/90]_S$, and $[\pm\theta/90_2]_S$, and $[\pm30/\pm60]_S$ families; solid line is the least square fit of all of the data points.

for each family of laminates, a unified relation between average interlaminar normal stress and interface moment can be established for all families of laminates. Figure 4 shows the results for all of the families plotted with the adjusted interlaminar normal stress $(\bar{\sigma}_z^{(k)}/\varepsilon_0)/K$ vs the interface moment $m^{(k)}/\varepsilon_0$. It can be seen that a very good linear relation exists between the two variables. The quantitative expression of this relation can be written as

$$\bar{\sigma}_z^{(k)} = 5.075Km^{(k)} \tag{5}$$

This equation describes a relation between $\bar{\sigma}_z^{(k)}$, which usually must be determined by finite element method or other analytical methods, and the interface moment m^k, which can easily be obtained from CLPT.

Prediction of Average Interlaminar Normal Stress and Onset of Open-Mode Delamination

Equation (5) can be used to predict average interlaminar normal stress in laminates of $[\pm\theta/0/90]_S$ or $[\pm\theta/90_2]_S$ families in conjunction with CLPT. As an example, average interlaminar normal stress in the $[\pm25/0/90]_S$ laminate family were predicted by Eq. (5). The predictions were compared with those calculated from the finite element model. It was found that the prediction of Eq. (5) is in fairly good agreement with the finite element results.

Interlaminar failure could be caused by interlaminar shear stresses, or normal stress, or both. Experimental data on the onset of delamination that is caused by interlaminar normal stress only are relatively rare in the literature. Open-mode delamination occurs at an interface where tensile interlaminar normal stress σ_z is predominant. For balanced symmetric laminates, those interfaces are usually in midplanes or near midplanes. It was found by Herakovich[8] that $[\pm45/0/90]_S$ laminate under tensile axial loading delaminated along the midplane where the interlaminar normal stress is the largest and interlaminar shear stresses are zero. It was observed in Ref. 9 that in the $[\pm25/90_2]_S$ laminate of T300/934 under tensile axial loading, delamination first occurs in the midplane.

Table 1 lists comparisons of prediction of the present method with existing experimental data in the literature on the axial stress corresponding to the onset of open-mode delamination for $[\pm45/0/90]_S$, $[\pm40/90_2]_S$, and $[\pm25/90_2]_S$ laminates. For these laminates, average interlaminar normal stress is assumed to be responsible for the onset of open-mode delamination. It is assumed that when the average interlaminar normal stress reaches the normal interlaminar strength, open-mode delamination occurs. For all material systems considered, experimental data on normal interlaminar strengths are

Table 1 Prediction and experimental data of axial stress corresponding to the onset of open-mode delamination for four laminates, megapascal

Laminate:	$[\pm45/0/90]_S$		$[\pm40/90_2]_S$	$[\pm25/90_2]_S$
Material	AS4/3501-6	T300/5208	T300/5208	T300/934
Experiment	452 (Ref. 10)	295 (Ref. 11)	163 (Ref. 12)	324 (Ref. 9)
	372[a] (Ref. 13)	313 (Ref. 12)		
Prediction	432	299	153	346

[a] AS/3501-6 data.

lacking and, thus, the transverse strength of the composite was taken as normal interlaminar strength. Generally, matrix cracking occurs before the onset of delamination. To account for the effect of in-plane matrix cracking, it is assumed that the normal interlaminar strength is 90% of the transverse strength when matrix cracking is present in adjacent laminas.[7,10] From Table 1, it is seen that the prediction of the present method is quite encouraging compared to Refs. 9–13.

Conclusions

A simplified method for predicting onset of open-mode free edge delamination has been proposed. The method takes advantage of a linear relation between average interlaminar normal stress and interface moment. Using this method, it is possible to predict open-mode free edge delamination by classical laminated plate theory for a class of laminates.

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